Real Analysis 1, MATH 5210, Fall 2022

Homework 12, Section 4.3 The Lebesgue Integral of a Measurable Nonnegative Function

Due Wednesday, November 30, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

4.17. Let E be a set of measure zero and define $f = \infty$ on E. Prove that $\int_E f = 0$.

4.18. Prove that the integral of a bounded measurable function of finite support is properly defined. That is, it is independent of the choice of a set E_0 of finite measure. HINT: Let E_1 and E_2 be measurable subsets of E such that $m(E_1) < \infty$, $m(E_2) < \infty$, $f \equiv 0$ on $E \setminus E_1$, and $f \equiv 0$ on $E \setminus E_2$. Notice that $E_1 = (E_1 \setminus E_2) \mathbin{\dot\cup} (E_1 \cap E_2)$ and $E_2 = (E_2 \setminus E_1) \mathbin{\dot\cup} (E_1 \cap E_2)$.

4.20. Let $\{f_n\}$ be a sequence of nonnegative measurable functions that converges to f pointwise on E. Let $M \geq 0$ be such that $\int_E f_n \leq M$ for all $n \in \mathbb{N}$.

(a) Prove that $\int_E f \leq M$.

(b) Prove that this property (i.e., part (a)) is equivalent to Fatou's Lemma. HINT: Let $M = \liminf\{\int_E f_n\}$ and let $\varepsilon > 0$. Show that $\int_E f_{n_k} < M + \varepsilon$ for some subsequence $\{\int_E f_{n_k}\}$ of $\{\int_E f_n\}$.