Real Analysis 1, MATH 5210, Fall 2022

Homework 2, Section 1.4 Borel Sets, Solutions

Due Saturday, September 3, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

Proposition 13. Let $\mathcal{C}$ be a collection of subsets of a set X. Then the intersection $\mathcal{A}$ of all σ-algebras of subsets of X that contain $\mathcal{C}$ is a σ-algebra and it is the smallest σ-algebra containing $\mathcal{C}$.

1.36. The collection of Borel sets is the smallest σ-algebra that contains all intervals of the form $[a, b)$ where $a < b$.

1.58(c). Let f be a continuous real-valued function on $\mathbb{R}$. The inverse image of a Borel set is Borel. HINT: For any function, $f^{-1}(\cup B_i) = \cup f^{-1}(B_i)$, $f^{-1}(\cap B_i) = \cap f^{-1}(B_i)$, and $f^{-1}(\mathbb{R} \setminus B) = \mathbb{R} \setminus f^{-1}(B)$. Consider $\mathcal{E}$ the collection of sets such that $E \in \mathcal{E}$ implies that $f^{-1}(E)$ is Borel. Show that $\mathcal{E}$ is a σ-algebra containing all open sets (so $\mathcal{E} \supset \mathcal{B}$).