Real Analysis 1, MATH 5210, Fall 2022

Homework 3, Section 2.1 Introduction,

Section 2.2 Lebesgue Outer Measure

Due Saturday, September 10, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

2.3. Let m' be a set function defined for all sets in a σ-algebra $\mathcal{A}$ with values in $[0,\infty]$. Assume m' is countably additive over countable disjoint collections of sets in $\mathcal{A}$. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of sets in $\mathcal{A}$. Prove that $m'(\cup_{k=1}^{\infty} E_k) \le \sum_{k=1}^{\infty} m'(E_k)$.

2.1.A. Let m' be a set function defined for all sets in a σ-algebra $\mathcal{A}$ with values in $[0,\infty]$. Assume m' is countably additive over countable disjoint collections of sets in $\mathcal{A}$. Let $\{E_k\}_{k=1}^{\infty}$ be a countable collection of sets in $\mathcal{A}$ where $E_k \subset E_{k+1}$ for all $k \in \mathbb{N}$. Prove that $m'(\cup_{k=1}^{\infty} E_k) = \lim_{k\to\infty} m'(E_k)$. This is called *continuity with respect to increasing sequences.*

2.6. Let A be the set of irrational numbers in the interval $[0,1]$. Prove that $m^*(A) = 1$.