Real Analysis 1, MATH 5210, Fall 2022

Homework 6, Section 2.3 The σ-Algebra of Lebesgue Measurable Sets and 2.4 Outer and Inner Approximation of Lebesgue Measurable Sets

Due Saturday, October 1, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

2.3.C. Let E be a measurable set with finite positive outer measure. Prove for any ε with $0 < \varepsilon < 1$ that there is a bounded interval I such that $\varepsilon m^*(I) \leq m^*(E \cap I) \leq m^*(I)$. HINT: Use the definition of outer measure and Theorem 0.3 with $\varepsilon > 0$ of the theorem replaced with $(1/\varepsilon - 1)m^*(E) > 0$. You will need Theorem 0.3 and countable additivity. Beware when you use bounded open intervals and disjoint open intervals.

2.17. Prove that a set E is measurable if and only if for each $\varepsilon > 0$, there is a closed set F and open set $\mathcal{O}$ for which $F \subseteq E \subseteq \mathcal{O}$ and $m^*(\mathcal{O} \setminus F) < \varepsilon$. HINT: Use Theorem 2.11.

2.18. (REVISED from the text's version.) Let $m^*(E) < \infty$. Then if there exists F_σ set F and G_δ set G with $F \subseteq E \subseteq G$ and $m^*(F) = m^*(E) = m^*(G)$, then E is measurable. NOTE: The text's statement is: "Let E have finite outer measure. Show that there is an F_σ set F and a G_δ set G such that $F \subseteq E \subseteq G$ and $m^*(F) = m^*(W) = m^*(G)$." This is incorrect since it does not assume that E is measurable. We know (from page 3 of the class notes for Section 2.3) that there exist F_σ set F and G_δ set G, called the inner approximation and outer approximation, such that $\lambda_*(F) = \lambda_*(E) \leq \lambda^*(E) = \lambda^*(G)$ (in terms of inner measure λ_* and outer measure λ^*). So the text's conclusion holds only if set E is measurable (which it did not assume). HINT: You may assume this behavior of inner and outer measure. Use the definition of Lebesgue measurable in terms of inner and outer measure.