Real Analysis 1, MATH 5210, Fall 2022

Homework 7, Section 2.5 Countable Additivity, Continuity, and the Borel-Cantelli Lemma, Solutions

2.24. Prove that if E_1 and E_2 are measurable then

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2).$$

HINT: Dissect $E_1 \cup E_2$ into disjoint measurable sets. Do not subtract measures of sets since this may give "$\infty - \infty$."

2.25. Show that the assumption that $m(B_1) < \infty$ is necessary in part (ii) of Theorem 2.15 (Continuity of Measure). That is, find a specific descending collection of sets $\{B_k\}_{k=1}^{\infty}$ with $m(B_1) = \infty$ such that

$$m\left(\cap_{k=1}^{\infty} B_k\right) \neq \lim_{k\to\infty} m(B_k).$$

2.26. Let $\{E_k\}_{k=1}^{\infty}$ be a countable disjoint collection of measurable sets. Prove that for any set A,

$$m^*\left(A \cap \left(\cup_{k=1}^{\infty} E_k\right)\right) = \sum_{k=1}^{\infty} m^*(A \cap E_k).$$

HINT: Use subadditivity, Proposition 2.6, and monotonicity.