Real Analysis 1, MATH 5210, Fall 2022

Homework 8, Section 2.6 Nonmeasurable Sets (partially from Royden's 3rd Edition), Solutions

Due Saturday, October 22, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

Problem 2.6.A. Show that if $E \in \mathcal{M}$ and $E \subset P$, then $m(E) = 0$. HINT: Let $E_i = E \overset{\circ}{+} r_i$, where $\mathbb{Q} \cap [0,1) = \{r_i\}_{i=1}^{\infty}$. Then $\{E_i\}_{i=1}^{\infty}$ is a disjoint sequence of measurable sets and $m(E_i) = m(E)$. Therefore $\sum m(E_i) = m(\cup E_i) \leq m([0,1))$.

Problem 2.6.B. Show that if A is any set with $m^*(A) > 0$, then there is a nonmeasurable set $E \subset A$. HINT: If $A \subset [0,1)$, let $E_i = A \cap P_i$. The measurability of E_i implies $m(E_i) = 0$, while $\sum m^*(E_i) \geq m^*(A) > 0$.

2.33. Let E be a nonmeasurable set of finite outer measure. Prove that there is a G_δ set G that contains E for which $m^*(E) = m^*(G)$, while $m^*(G \setminus E) > 0$.