Real Analysis 1, MATH 5210, Fall 2022

Homework 9, Section 3.1 Sums, Products, and Compositions

Due Saturday, October 29, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

3.3. Suppose a function f has a measurable domain E and is continuous except at a finite number of points. Is f necessarily measurable? If so, then prove it. If not, then give an example.

3.5. Suppose the function f is defined on a measurable set E and suppose f has the property that $\{x \in E \mid f(x) > c\} \in \mathcal{M}$ for each rational number c. Is f necessarily measurable? If so, then prove it. If not, then give an example.

3.6. Let f be a function with measurable domain D. Prove that f is measurable if and only if the function g defined on $\mathbb{R}$ by $g(x) = f(x)$ for $x \in D$ and $g(x) = 0$ for $x \notin D$ is measurable.

3.7. **(Bonus)** Let the function f be defined on a measurable set E. Prove that f is measurable if and only if for each Borel set A, $f^{-1}(A)$ is measurable.