# Real Analysis 1, MATH 5210, Spring 2017

## Homework 1, Sequential Pointwise Limits and Simple Approximation (3.2)

Due Friday, January 20, at 1:30

**Write in complete sentences!!!** *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

**3.12.** Let $f$ be a bounded measurable function on $E$. Prove there are sequences of simple functions on $E$, $\{\varphi_n\}$ and $\{\psi_n\}$, such that $\{\varphi_n\}$ is increasing and $\{\psi_n\}$ is decreasing and each of these sequences converges to $f$ uniformly. HINT: Use partitions and refinements of these partitions.

**3.14/15.**

**3.14.** Let $f$ be a measurable function on $E$ that is finite a.e. on $E$ and $m(E) < \infty$. Prove that for each $\varepsilon > 0$, there is a measurable set $F$ contained in $E$ such that $f$ is bounded on $F$ and $m(E \setminus F) < \varepsilon$.

**3.15.** Let $f$ be a measurable function on $E$ that is finite a.e. on $E$ and $m(E) < \infty$. Prove that for each $\varepsilon > 0$ there is a measurable set $F$ contained in $E$ and a sequence $\{\varphi_n\}$ of simple functions on $E$ such that $\{\varphi_n\} \to f$ uniformly on $F$ and $m(E \setminus F) < \varepsilon$. HINT: Use Exercises 3.12 and 3.14.

**3.21.** For a sequence $\{f_n\}$ of measurable functions with common domain $E$, prove that each of the following functions is measurable: $\inf\{f_n\}$, $\sup\{f_n\}$, $\liminf\{f_n\}$, and $\limsup\{f_n\}$. HINT: Use the definition of measurable on $\inf\{f_n\}$ and $\sup\{f_n\}$. We have $\overline{\lim} f_n(x) = \inf_{n\in\mathbb{N}} \left\{\sup_{k\geq n}\{f_k(x)\}\right\}$ and $\underline{\lim} f_n(x) = \sup_{n\in\mathbb{N}} \left\{\inf_{k\geq n}\{f_k(x)\}\right\}$.