Real Analysis 1, MATH 5210, Spring 2017

Homework 11, Measures and Measurable Sets (17.1)

Due Friday, April 14, at 1:40

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

17.5. Let $(X, \mathcal{M}, \mu)$ be a measure space. the symmetric difference, $E_1 \triangle E_2$, of two subsets E_1 and E_2 of X is defined as $E_1 \triangle E_2 = (E_1 \setminus E_2) \mathbin{\dot\cup} (E_2 \setminus E_1)$.
(i) Show that if $E_1, E_2 \in \mathcal{M}$ and $\mu(E_1 \triangle E_2) = 0$, then $\mu(E_1) = \mu(E_2)$.
(ii) Show that if μ is complete and $E_1 \in \mathcal{M}$, then $E_2 \in \mathcal{M}$ if $\mu(E_1 \triangle E_2) = 0$.
HINT: For (ii), we have $E_1 \setminus (E_1 \setminus E_2) = E_1 \cap E_2$ and $E_2 = (E_2 \setminus E_1) \mathbin{\dot\cup} (E_1 \cap E_2)$.

17.7. Let $(X, \mathcal{M})$ be a measurable space.
(i) If μ and ν are measures defined on $\mathcal{M}$, then the set function λ defined on $\mathcal{M}$ by $\lambda(E) = \mu(E) + \nu(E)$ also is a measure, denoted $\lambda = \mu + \nu$.

(ii) If μ and ν are measures on $\mathcal{M}$ and $\mu \geq \nu$ then there is a measure λ on $\mathcal{M}$ for which $\mu = \nu + \lambda$. HINT: Define

$$\lambda(E) = \begin{cases} \mu(E) - \nu(E) & \text{if } \nu(E) < \infty \\ \sup\{\mu(F) - \nu(F)\} & \text{if } \nu(E) = \infty \end{cases}$$

where the supremum is taken over all $F \in \mathcal{M}$, $F \subset E$, and $\nu(E) < \infty$. Show (1) $\mu = \nu + \lambda$ on $\mathcal{M}$, (2) λ is finitely additive, and (3) λ is countably additive.