Real Analysis 1, MATH 5210, Spring 2017

Homework 12, Signed Measures (17.2)

Due Friday, April 21, at 1:40

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

17.13. (Modified) Prove the uniqueness claim of the Jordan Decomposition Theorem.

17.14. Let ν be a signed measure and let $\nu = \nu^+ - \nu^-$ be the Jordan decomposition of ν. Prove that if E is any measurable set, then $-\nu^-(E) \leq \nu(E) \leq \nu^+(E)$ and $|\nu(E)| \leq |\nu|(E)$.

17.15. Let ν_1 and ν_2 be finite signed measures (that is, signed measures such that every measurable set E satisfies $|\nu_1(E)| < \infty$ and $|\nu_2(E)| < \infty$). Prove for all $\alpha, \beta \in \mathbb{R}$ that $\alpha\nu_1 + \beta\nu_2$ is a finite signed measure. Prove that $|\alpha\nu_1| = |\alpha||\nu_1|$ and $|\nu_1 + \nu_2| \leq |\nu_1| + |\nu_2|$, where "$\nu \leq \mu$" means that $\nu(E) \leq \mu(E)$ for all measurable sets E.