Real Analysis 1, MATH 5210, Spring 2017

Homework 3, Convergence in Measure (5.2) and Convex Functions (6.6)

Due Friday, February 3, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

5.8a. Fatou's Lemma holds for convergence in measure: Let $\{f_n\}$ be a sequence of nonnegative measurable functions on E. If $\{f_n\} \to f$ in measure, then

$$\int_E f \leq \liminf \left(\int_E f_n \right).$$

HINT: Consider a subsequence of $\{\int_E f_n\}_{n=1}^{\infty}$ which converges to $\liminf(\int_E f_n)$. Apply Theorem 5.4 to the subsequence and consider the associated integrals.

5.8b. The Monotone Convergence Theorem holds for convergence in measure: Let $\{f_n\}$ be an increasing sequence of nonnegative measurable functions on E. If $\{f_n\} \to f$ in measure on E, then

$$\lim_{n\to\infty} \left(\int_E f_n \right) = \int_E \left(\lim_{n\to\infty} f_n \right) = \int_E f.$$

6.67. State and prove a version of Jensen's Inequality on a general closed, bounded interval $[a, b]$. HINT: In the proof of Jensen's Inequality, replace the supporting line at α with a supporting line at $\frac{\alpha}{b-a}$ where $\alpha = \int_a^b f(x)\, dx$.