Real Analysis 1, MATH 5210, Spring 2017

Homework 5, The Inequalities of Young, Holder, and Minkowski (7.2); Solutions

Due Friday, February 17, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

7.12. For $1 \le p < \infty$ and a sequence $a = (a_1, a_2, \ldots) \in \ell^p$, define T_a to be the function on the interval $[1, \infty)$ that takes the value a_k on $[k, k+1)$, for $k = 1, 2, \ldots$. In parts (a) and (b) we give a proof of Hölder's Inequality for ℓ^p using Hölder's Inequality in $L^p[1, \infty)$.

(a) Show that $T_a \in L^p[1, \infty)$ and $\|a\|_p = \|T_a\|_p$.

(b) Prove a Hölder's Inequality for ℓ^p.

7.12 (cont.) In parts (c) and (d) we give a proof of Minkowski's Inequality for ℓ^p using Minkowski's Inequality in $L^p[1, \infty)$.

(c) For $a \in \ell^p$, define a^*, show $a^* \in \ell^q$, and that $\sum_{k=1}^{\infty} a_k a_k^* = \|a\|_p$.

(d) Prove a Minkowski Inequality for ℓ^p. NOTE: The Minkowski Inequality allows us to conclude that ℓ^p is a normed linear space.

7.18. Assume $m(E) < \infty$. For $f \in L^\infty(E)$, show that $\lim_{p\to\infty} \|f\|_p = \|f\|_\infty$. HINT: First show that $\limsup_{p\to\infty} \|f\|_p \le \|f\|_\infty$. Second, let $\varepsilon > 0$ and define $A = \{x \in E \mid |f| \ge \|f\|_\infty - \varepsilon\}$. Show that $\liminf_{p\to\infty} \|f\|_p \ge \|f\|_\infty - \varepsilon$.