Real Analysis 1, MATH 5210, Spring 2017

Homework 9, Groups, Fields, Vector Spaces; Inner Product Spaces (HWG5.1, HWG5.2)

Due Friday, March 31, at 1:40

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

5.1.3. If B_1 and B_2 are Hamel bases for a given infinite dimensional vector space, then B_1 and B_2 are of the same cardinality. HINT: You need two results from set theory. Hungerford's *Algebra* (1974) page 17 gives:

Theorem 8.6. The Schroeder-Bernstein Theorem. If A and B are sets such that $|a| \leq |B|$ and $|B| \leq |A|$, then $|A| = |B|$.

Hungerford's *Algebra* (1974) page 22 gives:

Exercise 0.8.11. If J is an infinite set, and for each $i \in J$ set A_j is a finite set, then $|\cup_{j \in J} A_j| \leq |J|$.

5.2.5. (Modified)

(a) Prove that equality holds in The Schwarz Inequality (Theorem 5.2.1), $|\langle \mathbf{u}, \mathbf{v} \rangle| = \|\mathbf{u}\|\|\mathbf{v}\|$, if and only if $\mathbf{u}$ is a scalar multiple of $\mathbf{v}$.

(b) Equality holds in the Triangle Inequality (Theorem 5.2.2), $\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$, if and only if $\mathbf{u}$ is a nonnegative scalar multiple of $\mathbf{v}$.

(c) Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in an inner product space. Prove that $\|\mathbf{u} - \mathbf{v}\| + \|\mathbf{v} - \mathbf{w}\| = \|\mathbf{u} - \mathbf{w}\|$ if and only if $\mathbf{v} = t\mathbf{u} + (1 - t)\mathbf{w}$ for some $t \in [0, 1]$.