Real Analysis 1, MATH 5210, Spring 2019

Homework 1, Convergence in Measure (5.2)

Due Friday, January 18, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

5.6. Let $\{f_n\} \to f$ in measure on E and let g be a measurable function on E that is finite a.e. on E. Then $\{f_n\} \to g$ in measure on E if and only if $f = g$ a.e. on E. NOTE: This result shows that the limit in measure of a sequence of functions in unique (up to "a.e."). HINT: The proof of $f = g$ a.e. implies $\{f_n\} \to g$ in measure is easy. To show that $f = g$ a.e. if convergence in measure holds, do by contradiction. Suppose $E_0 = \{x \in E \mid f(x) \neq g(x)\}$ satisfies $m(E_0) > 0$. Write E_0 as a union of an ascending sequence of sets and use Continuity of Measure to find a set of positive measure on which $|f(x) - g(x)| > K$ for some fixed $K > 0$. Use this set to show that either $\{f_n\}$ does not converge to f in measure or that $\{f_n\}$ does not converge to g in measure.

5.8(b). The Monotone Convergence Theorem holds for convergence in measure: Let $\{f_n\}$ be an increasing sequence of nonnegative measurable functions on E. If $\{f_n\} \to f$ in measure on E, then

$$\lim_{n\to\infty} \left(\int_E f_n \right) = \int_E \left(\lim_{n\to\infty} f_n \right) = \int_E f.$$

5.9. Show that Proposition 3 does not necessarily hold for sets E of infinite measure.