Real Analysis 1, MATH 5210, Spring 2019

Homework 11, The Construction of Outer Measures (17.4), The Caratheodory-Hahn Theorem (17.5)

Due Thursday, April 25, at 2:15

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

17.18. Let $\mu^* : 2^X \to [0, \infty]$ be an outer measure. Let $A \subset X$, $\{E_k\}_{k=1}^\infty$ a disjoint countable collection of measurable sets and $E = \cup_{k=1}^\infty E_k$. Then $\mu^*(A \cap E) = \sum_{k=1}^\infty \mu^*(A \cap E_k)$. NOTE: This is an extension of Proposition 17.6 from finite collections of E_k to countably infinite collections of E_k.

17.25. Let X be any set containing more than one point and A a proper nonempty subset of X. Define $\mathcal{S} = \{A, X\}$ and the set function $\mu : \mathcal{S} \to [0, \infty]$ by $\mu(A) = 1$ and $\mu(X) = 2$.

(i) Show that $\mu : \mathcal{S} \to [0, \infty]$ is a premeasure.

(ii) Can μ be extended to a measure?

(iii) (Bonus) What are the subsets of X that are measurable with respect to the outer measure μ^* induced by μ? HINT: The answer is $\varnothing$ and X.