Real Analysis 1, MATH 5210, Spring 2019

Homework 2, Convex Functions (6.6)

Due Friday, January 25, at 1:30

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

Lemma 6.16. (6.6.A.) Let φ be a convex function on (a, b). Then φ has left-hand and right-hand derivatives at each point $x \in (a, b)$. Moreover, for points $u, v \in (a, b)$ with $u < v$ these one-sided derivatives satisfy the following inequality:

$$\varphi'(u^-) \leq \varphi'(u^+) \leq \frac{\varphi(v) - \varphi(u)}{v - u} \leq \varphi'(v^-) \leq \varphi'(v^+).$$

HINT: Use the Chordal Slope Lemma.

6.68. Let f be integrable over $[0, 1]$. Prove that

$$\exp\left(\int_{[0,1]} f \right) \leq \int_{[0,1]} \exp(f).$$

HINT: You do not know that $\exp(f)$ is integrable on $[0, 1]$.