Real Analysis 1, MATH 5210, Spring 2019

Homework 4, The Inequalities of Young, Holder, and Minkowski (7.2)

Due Tuesday, February 12, at 2:15

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

7.7(a). Let $E = (0, 1]$ and let $1 \leq p_1 < p_2 \leq \infty$. Show that $f(x) = x^\alpha$, where $-1/p_1 < \alpha < -1/p_2$, satisfies $f \in L^{p_1}(E)$ but $f \notin L^{p_2}(E)$.

7.8. Let $f, g \in L^2(E)$. From the linearity of integration show that for any $\lambda \in \mathbb{R}$,

$$\lambda^2 \int_E |f|^2 + 2\lambda \int_E |fg| + \int_E |g|^2 = \int_E (\lambda|f| + |g|)^2 \geq 0.$$

From this and the quadratic formula directly derive the Cauchy-Schwarz Inequality.

7.18. Assume $m(E) < \infty$. For $f \in L^\infty(E)$, show that $\lim_{p\to\infty} \|f\|_p = \|f\|_\infty$. HINT: First show that $\limsup_{p\to\infty} \|f\|_p \leq \|f\|_\infty$. Second, let $\varepsilon > 0$ and define $A = \{x \in E \mid |f| \geq \|f\|_\infty - \varepsilon\}$. Show that $\liminf_{p\to\infty} \|f\|_p \geq \|f\|_\infty - \varepsilon$.