Real Analysis 1, MATH 5210, Spring 2019

Homework 5, L^p is Complete: The Riesz-Fischer Theorem (7.3)

Due Tuesday, February 19, at 2:15

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

7.25. Let E have finite measure and $1 \leq p_1 < p_2 \leq \infty$. If $\{f_n\} \to f$ in $L^{p_2}(E)$ (i.e., with respect to the L^{p_2} norm) then $\{f_n\} \to f$ in $L^{p_1}(E)$.

7.26. The L^p Lebesgue Dominated Convergence Theorem.
Let $\{f_n\}$ be a sequence of measurable functions that converge pointwise a.e. on E to f. For $1 \leq p < \infty$, suppose there is a function $g \in L^p(E)$ such that for all $n \in \mathbb{N}$, $|f_n| \leq g$ a.e. on E. Then $\{f_n\} \to f$ in $L^p(E)$.

7.29. Consider the linear space of polynomials on $[a, b]$ normed by $\|\cdot\|_{\max}$ norm. Is this normed linear space a Banach space?