Real Analysis 1, MATH 5210, Spring 2019

Homework 6, Approximation and Separability (7.4)

Due Tuesday, February 26, at 2:15

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

7.36. Let $\mathcal{S}$ be a subset of a normed linear space X. Prove that $\mathcal{S}$ is dense in X if and only if each $g \in X$ is the limit of a sequence in $\mathcal{S}$.

7.37. Let X be a normed linear space with $\mathcal{F} \subset \mathcal{G} \subset \mathcal{H} \subset X$. Prove that if $\mathcal{F}$ is dense in $\mathcal{G}$ and if $\mathcal{G}$ is dense in $\mathcal{H}$, then $\mathcal{F}$ is dense in $\mathcal{H}$.

7.43. Suppose X is a Banach space with norm $\|\cdot\|$. Let X_0 be a dense subspace of X. Assume that X_0 when normed by the norm it inherits from X, is also a Banach space. Prove that $X = X_0$. NOTE: This result holds in a complete metric space. The proof does not require the "linear" part of "normed linear space."

7.44. **(Bonus)** For $1 \leq p < \infty$, show that ℓ^p is separable. Show that ℓ^∞ is not separable.