Real Analysis 1, MATH 5210, Spring 2019

Homework 7, The Riesz Representation for the Dual of L^p, $1 \leq p < \infty$ (8.1)

Due Thursday, March 7, at 2:15

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

8.1. For T a bounded linear functional on linear space X, use the definition of $\|T\|_*$ to prove that

$$\|T\|_* = \sup\{|T(f)| \mid f \in X, \|f\| = 1\}.$$

8.2. Prove Proposition 8.1: Let X be a normed linear space. Then the collection of bounded linear functionals on X is a linear space on which $\|\cdot\|_*$ is a norm. This normed linear space is the *dual space* of X, denoted X^*.

8.3(b). Let T be a linear functional on a normed linear space X. Prove that T is bounded if and only if for all convergent sequences in X, say $\{f_n\} \to f$ in X, we have $\{T(f_n)\} \to T(f)$ in $\mathbb{R}$. HINT: You may assumer part (a): "Let T be a linear functional on a normed linear space X. Suppose that for all sequences $\{f_n\}_{n=1}^{\infty}$ in X with $\{f_n\} \to f$ where $f \in X$ we have $\{T(f_n)\} \to T(f)$ in $\mathbb{R}$. Prove that T is continuous on X." Use part (a) and continuity at 0 for the "if" part.