Real Analysis 1, MATH 5210, Spring 2019

Homework 8, Groups, Fields, and Vector Spaces; Inner Product Spaces (HWG 5.1, 5.2)

Due Tuesday, March 26, at 2:15

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

5.1.7. Consider the vector space of functions continuous on the interval $[a, b]$, denoted $C([a, b])$. Show that $f_1(x) = \cos x$ and $f_2(x) = \sin x$ are linearly independent in this space. Show that the set $\{1, x, x^2, \ldots, x^n\}$ is a linearly independent set in $C([a, b])$.

5.2.6. Prove that the inner product if continuous. That is, if $(\mathbf{u}_n) \to \mathbf{u}$ and $(\mathbf{v}_n) \to \mathbf{v}$, then $\langle \mathbf{u}_n, \mathbf{v}_n \rangle \to \langle \mathbf{u}, \mathbf{v} \rangle$.

5.2.5(a) Prove that equality holds in The Schwarz Inequality (Theorem 5.2.1), $|\langle \mathbf{u}, \mathbf{v} \rangle| = \|\mathbf{u}\|\|\mathbf{v}\|$, if and only if $\mathbf{u}$ is a scalar multiple of $\mathbf{v}$.

5.2.5(b) (Bonus) Equality holds in the Triangle Inequality (Theorem 5.2.2), $\|\mathbf{u}+\mathbf{v}\| \leq \|\mathbf{u}\|+\|\mathbf{v}\|$, if and only if $\mathbf{u}$ is a nonnegative scalar multiple of $\mathbf{v}$.

5.2.5(c) (Bonus) Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in an inner product space. Prove that $\|\mathbf{u} - \mathbf{v}\| + \|\mathbf{v} - \mathbf{w}\| = \|\mathbf{u} - \mathbf{w}\|$ if and only if $\mathbf{v} = t\mathbf{u} + (1 - t)\mathbf{w}$ for some $t \in [0, 1]$.