Real Analysis 2, MATH 5220, Spring 2023

Homework 2, 6.6. Convex Functions

Due Saturday, January 21, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

Lemma 6.16. (6.6.A.) Let φ be a convex function on (a, b). Then φ has left-hand and righthand derivatives at each point $x \in (a, b)$. Moreover, for points $u, v \in (a, b)$ with u < v these one-sided derivatives satisfy the following inequality:

$$\varphi'(u^-) \le \varphi'(u^+) \le \frac{\varphi(v) - \varphi(u)}{v - u} \le \varphi'(v^-) \le \varphi'(v^+).$$

HINT: Use the Chordal Slope Lemma.

6.68. Let f be integrable over [0, 1]. Prove that

$$\exp\left(\int_{[0,1]} f\right) \le \int_{[0,1]} \exp(f).$$

HINT: You do not know that $\exp(f)$ is integrable on [0, 1].