Real Analysis 2, MATH 5220, Spring 2023

Homework 10, Section 5.4. Projections and Hilbert Space Isomorphisms (of Hong, Wang, Gardner)

Due Saturday, April 8, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

5.4.2. Prove that if S is an orthonormal set in an inner product space, then any finite subset of S is linearly independent.

5.4.6. Prove that $R = \{(1, 0, 0, \ldots), (0, 1, 0, \ldots), \ldots\} \subset \ell^2$ is a (topologically) closed set and a bounded set, but not a compact set. NOTE: Recall that the Heine-Borel Theorem (Theorem 1.4.11) states that a set in $\mathbb{R}^n$ is compact if and only if it is closed and bounded. The example given here shows that the familiar Heine-Borel Theorem does not hold in all metric spaces. Also notice that R is an infinite bounded set with no limit points, indicating that the Bolzano-Weierstrass Theorem (see Exercise 3 of Section 1.4) does not hold in ℓ^2.