Real Analysis 2, MATH 5220, Spring 2023

Homework 11, Section 9.1. Examples of Metric Spaces, Solutions

Due Saturday, April 15, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

9.1. Prove that two metrics ρ and τ on the same set X are equivalent if and only if there is a $c > 0$ such that for all $u, v \in X$ we have

$$\frac{1}{c}\tau(u, v) \leq \rho(u, v) \leq c\tau(u, v).$$

9.4. For a closed and bounded interval $[a, b]$, consider the set $X = C[a, b]$ of continuous real-valued functions on $[a, b]$. Prove that the metric induced by the maximum norm and the metric induced by the $L^1[a, b]$ norm are not equivalent.