Real Analysis 2, MATH 5220, Spring 2023

Homework 3, 7.2. The Inequalities of Young, Holder, and Minkowski

Due Saturday, February 4, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

7.13. Prove that if f is a bounded function on E that belongs to $L^{p_1}(E)$, then f belongs to $L^{p_2}(E)$ for any $p_2 > p_1$. HINT: Define $E_1 = \{x \in E \mid |f(x)| \leq 1\}$ and $E_\infty = E \setminus E_1$.

7.18. Assume $m(E) < \infty$. For $f \in L^\infty(E)$, show that $\lim_{p\to\infty} \|f\|_p = \|f\|_\infty$. HINT: First show that $\limsup_{p\to\infty} \|f\|_p \leq \|f\|_\infty$. Second, let $\varepsilon > 0$ and define $A = \{x \in E \mid |f| \geq \|f\|_\infty - \varepsilon\}$. Show that $\liminf_{p\to\infty} \|f\|_p \geq \|f\|_\infty - \varepsilon$.