Real Analysis 2, MATH 5220, Spring 2023

Homework 4, 7.3. L^p Is Complete: The Riesz-Fischer Theorem

Due Saturday, February 11, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

7.26. The L^p Lebesgue Dominated Convergence Theorem.

Let $\{f_n\}$ be a sequence of measurable functions that converge pointwise a.e. on E to f. For $1 \leq p < \infty$, suppose there is a function $g \in L^p(E)$ such that for all $n \in \mathbb{N}$, $|f_n| \leq g$ a.e. on E. Then $\{f_n\} \to f$ in $L^p(E)$.

7.34. **(b)** Prove that the ℓ^p spaces are Banach spaces for $1 \leq p \leq \infty$. This is the Riesz-Fischer Theorem for ℓ^p. You may assume Exercise 7.34(a): Let $1 \leq p \leq \infty$. Every rapidly Cauchy sequence in ℓ^p converges with respect to the ℓ^p norm to an element of ℓ^p.