Real Analysis 2, MATH 5220, Spring 2023

Homework 5, 7.4. Approximation and Separability

Due Saturday, February 18, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

7.43. Suppose X is a Banach space with norm $\|\cdot\|$. Let X_0 be a dense subspace of X. Assume that X_0 when normed by the norm it inherits from X, is also a Banach space. Prove that $X = X_0$. NOTE: This result holds in a complete metric space. The proof does not require the "linear" part of "normed linear space."

7.44(a). For $1 \leq p < \infty$, show that ℓ^p is separable.