

Real Analysis 2, MATH 5220, Spring 2023

Homework 6, 8.1. The Riesz Representation for the Dual of

$$L^p, 1 \leq p < \infty$$

Due Saturday, February 25, at 11:59 p.m.

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **do your own work!!!**

8.1. For T a bounded linear functional on linear space X , use the definition of $\|T\|_*$ to prove that

$$\|T\|_* = \sup\{|T(f)| \mid f \in X, \|f\| = 1\}.$$

8.2. Prove Proposition 8.1: Let X be a normed linear space. Then the collection of bounded linear functionals on X is a linear space on which $\|\cdot\|_*$ is a norm. This normed linear space is the *dual space* of X , denoted X^* .