Real Analysis 1, Test 1 Study Guide

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The Riemann-Lebesgue Theorem. Partition, Riemann sums, upper and lower Riemann integrals, Riemann integral, conditions for Riemann integrability, Riemann integrability of continuous functions, measure of an open interval, measure zero set, countable sets are measure zero, oscillation of a function, oscillation and continuity, the Riemann-Lebesgue Theorem Part a, $A_s = \{x \in [a,b] \mid \operatorname{osc}(f;x) \geq s\}$ is compact, the Riemann-Lebesgue Theorem Part b, Dirichlet function, uniform convergence of a sequence of functions, Riemann integrals of uniformly convergent series of functions.

1.4 Open Sets, Closed Sets, and Borel Sets. Definition of the real numbers ("a complete ordered field") and the Axiom of Completeness, Algebra of sets, algebra generated by a collection of sets, σ-algebra of sets, σ-algebra generated by a collection of sets, Borel sets, cardinality of the collection of Borel sets; F_σ sets, G_δ sets, $F_{\sigma\delta}$ sets, $G_{\delta\sigma}$ sets, etc.; Young's Theorem (the set of points on which a function is continuous).

2.1 Introduction (to Lebesgue Measure). The four desired properties of a measure, monotonicity, translation invariance, countable subadditivity.

2.2 Lebesgue Outer Measure. Outer measure, outer measure of an interval (Proposition 2.1), outer measure is translation invariant (Proposition 2.2), outer measure is countably subadditive (Proposition 2.3), approximation of a bounded set with a G_δ set (Exercise 2.7).

2.3 The σ-Algebra of Lebesgue Measurable Sets. (Lebesgue) measurable sets and the Carathéodory splitting condition, the inner/outer measure approach to Lebesgue measurable sets (measure of a closed and bounded set, inner measure, countable superadditivity of inner measure, outer approximation [outer content or measurable cover], inner approximation [inner content or measurable kernal], definition of "measurable" in terms of inner and outer measure), outer measure zero sets are measurable (Proposition 2.4), the union of a finite collection of measurable sets is measurable (Proposition 2.5), finite additivity of outer measure (Proposition 2.6), the union of a countable collection of measurable sets is measurable (Proposition 2.7), outer measure is countably additive (Proposition 2.13), intervals are measurable (Proposition 2.8), the translate of a measurable set is measurable (Proposition 2.10).

2.4 Outer and Inner Approximation of Lebesgue Measurable Sets. Properties equivalent to the measurability of a set and inner and outer approximation (Theorem 2.11).

2.5 Countable Additivity, Continuity, and the Borel-Cantelli Lemma. Lebesgue measure, Measure in Continuous (Theorem 2.15), almost everywhere, Borel-Cantelli Lemma.