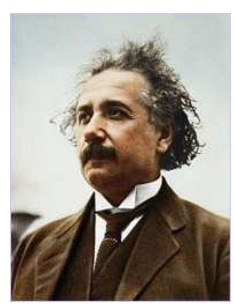
Chapter 1. Surfaces and the Concept of Curvature

Note. In this chapter, we define the concept of curvature and physically motivate it in the setting of a path (a 1-dimensional manifold) and a surface (a 2-dimensional manifold). The work involving surfaces is due to Carl Friederich Gauss (1777–1855) and is presented in Sections 1.2 to 1.8. Gauss's student, Georg Friederich Bernhard Riemann (1826–1866), then extends Gauss's concepts to general n-dimensional manifolds in his work of the 1850s (which is lightly covered in Section 1.9). After presenting his special theory of relativity (which we cover in Chapter 2), Albert Einstein (1879–1955) learns the techniques of Riemann to give his general theory of relativity (which we cover in Chapter 3) based on a curved spacetime in 1915 (published in 1916).

Carl Gauss

Georg Riemann



Albert Einstein

Notation. We shall denote the familiar three dimensional Euclidean space (traditionally denoted $\mathbb{R}^3$) as E^3.

Recall. The Euclidean norm on E^3 is

$$\|\vec{x}\| = \|(x, y, z)\| = \sqrt{x^2 + y^2 + z^2}.$$

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