

3.1 The Principle of Equivalence

Note. Newton's Second Law of Motion ($\vec{F} = m\vec{a}$) treats "mass" as an objects resistance to changes in movement (or *acceleration*). This is an objects *inertial mass*. In Newton's Law of Universal Gravitation ($F = GMm/r^2$), an objects mass measures its response to *gravitational attraction* (called its *gravitational mass*). Einstein was bothered by the dichotomy in the idea of mass:

inertial mass
acceleration

gravitational mass
gravitational acceleration

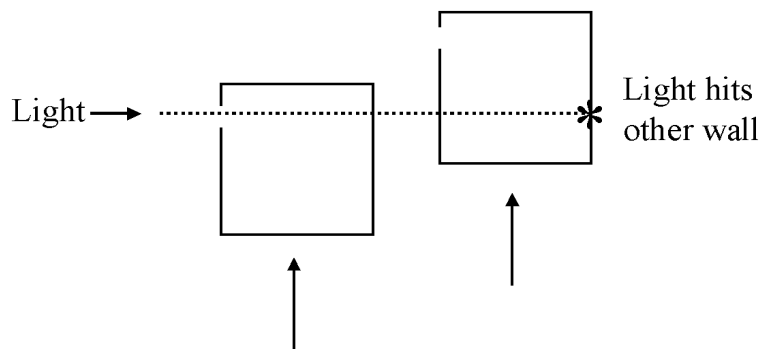
As we'll see, he resolved this by putting gravity and acceleration on an equivalent footing. This is discussed in Chapters 19 and 20 of Einstein's *Relativity: The Special and General Theory* (translated by Robert W. Lawson), NY: Three Rivers Press (1961).

Note. Consider an observer in a sealed box. First, if this box is in free fall in a gravitational field, then the observer in the box will think that he is weightless in an inertial (unaccelerating) frame. There is no experiment he can perform (entirely within the box) to detect the acceleration or the presence of the gravitational field. Second, if the observer is out in deep space and under no gravitational influence BUT is accelerating rapidly, then he will interpret the acceleration as the presence of a gravitational field. Again, there is no experiment he can perform (within the box) which will reveal that he is accelerating rapidly, versus being stationary with respect to a gravitational field. Einstein resolved these observations (and the inertial versus gravitational mass question) in the Principle of Equivalence.

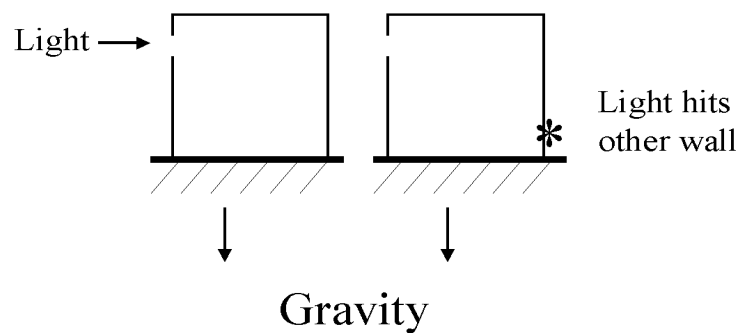
Definition. Principle of Equivalence.

There is no way to distinguish between the effects of acceleration and the effects of gravity - they are equivalent.

Note. A consequence of the Principle of Equivalence is that light is “bent” when in a gravitational field. Consider the observer in an accelerating box. If a ray of light enters a hole in one side of the box, it hits the other side of the box at a point slightly lower:



Now consider an observer in a box under the influence of gravity. By the Principle of Equivalence, he must observe the same thing:



Example. Consider a collection of n particles of masses m_1, m_2, \dots, m_n which interact (gravitationally, say) with force \vec{F}_{ij} between particle i and j (and so $\vec{F}_{ij} = -\vec{F}_{ji}$) for $i \neq j$. Suppose observer #1 uses coordinates x, y, z, t and observer #2 uses coordinates x', y', z', t' . We assume low relative velocities and so $t = t'$. Let \vec{X}_i be the location of particle i in observer #1's coordinates and let \vec{X}'_i be the location of particle i in observer #2's coordinates. Therefore we use $\vec{X} = (x, y, z)$ and $\vec{X}' = (x', y', z')$ for the spatial coordinate vectors. Suppose observer #1 *believes* that he and the particles are in the presence of a gravitational field \vec{g} (and interprets that he is stationary experiencing the gravity and the particles are in free fall) and so he sees all particles moving away from him with acceleration \vec{g} . The equations of motion for the particles for observer #1 are

$$m_i \frac{d^2 \vec{X}_i}{dt^2} = m_i \vec{g} + \sum_{j \neq i} \vec{F}_{ji} \quad (*)$$

for $i = 1, 2, \dots, n$.

Next, Suppose observer #2 is moving relative to observer #1 in such a way that $\vec{X}' = \vec{X} - \frac{1}{2} \vec{g} t^2$ (Frame #2 is accelerating relative to Frame #1 and conversely). By differentiating twice with respect to t :

$$\frac{d^2 \vec{X}'}{dt^2} = \frac{d^2 \vec{X}}{dt^2} - \vec{g}$$

or

$$\frac{d^2 \vec{X}}{dt^2} = \frac{d^2 \vec{X}'}{dt^2} + \vec{g}.$$

Therefore, for the i th particle

$$\frac{d^2 \vec{X}_i}{dt^2} = \frac{d^2 \vec{X}'_i}{dt^2} + \vec{g} \text{ or } m_i \frac{d^2 \vec{X}_i}{dt^2} = m_i \frac{d^2 \vec{X}'_i}{dt^2} + m_i \vec{g}.$$

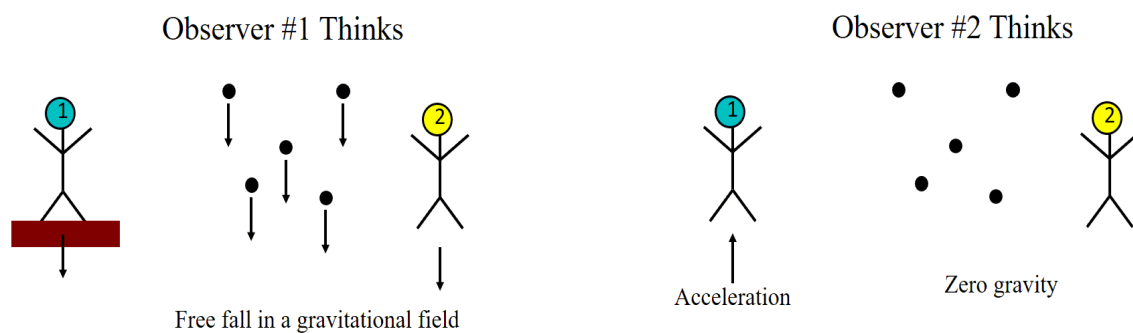
Substituting into equation (*) gives

$$m_i \frac{d^2 \vec{X}'_i}{dt^2} = \sum_{j \neq i} \vec{F}_{ji}.$$

Therefore, the gravitational field has been “transformed” away!

- Observer #1 thinks that observer #2 (along with the particles) is in free fall and that is why he does not “see” the gravitational field.
- Observer #2 thinks there is no gravitational field. He thinks that he is an inertial observer and that observer #1 is accelerating away from observer #2 and the particles.

The Principle of Equivalence states that both observers are “right.” Therefore the Principle of Equivalence puts all frames (inertial or not) on the “same footing.” In particular, gravitational force is equivalent to a force created by acceleration.



Revised: 6/30/2019