Write in complete sentences!!! Explain what you are doing and convince me that you understand what you are doing and why.

I.1.6(a) Show that the plane curve \( \vec{\alpha}(t) = (x(t), y(t)) \) has curvature
\[
k(t) = \frac{|x'(t)y''(t) - x''(t)y'(t)|}{(x'(t)^2 + y'(t)^2)^{3/2}}
\]
at \( \vec{\alpha}(t) \). Notice from Exercise I.1.5 that we have \( k(t) = \frac{\|d\vec{T}/dt\|}{\|\vec{\alpha}'(t)\|} \) and \( \vec{T} = \frac{\vec{\alpha}'(t)}{\|\vec{\alpha}'(t)\|} \).

I.1.6(b) As a special case, show that the graph of \( y = f(x) \) has curvature
\[
k(x) = \left| \frac{f''(x)}{(1 + f'(x)^2)^{3/2}} \right|
\]
at \((x, f(x))\).

I.1.7(a) Let \( \vec{\alpha}(t) = (a \cos t, b \sin t) \) for \( 0 \leq y \leq 2\pi \). Since \( x^2/a^2 + y^2/b^2 = 1 \), the image of \( \vec{\alpha} \) is an ellipse. Compute its curvature \( k(t) \) by the formula of Exercise I.1.6(a) at \( t = 0 \) and \( t = \pi/2 \).

I.1.7(b) Sketch the ellipse \( x^2/4 + y^2 = 1 \) and its osculating circles at the points \((2, 0)\) and \((0, 1)\). Find the equations of the osculating circles.

I.1.9. Let \( \vec{\alpha}(t) \) be a smooth curve in \( E^3 \), where \( t \) is an arbitrary parameter. Let \( v(t) = ds/dt \) be the speed at parameter value \( t \). Then
\[
\vec{\alpha}'(t) = \frac{d\vec{\alpha}}{ds} \frac{ds}{dt} = v\vec{T} \text{ and } \vec{T}'(t) = \frac{d\vec{T}}{ds} \frac{ds}{dt} = kv\vec{N}.
\]
Prove that
\[
k = \frac{\|\vec{\alpha}' \times \vec{\alpha}''\|}{\|\vec{\alpha}'\|^3}
\]
where the primes indicate derivatives with respect to \( t \).

I.1.10. Compute the curvature of the helix \( \vec{\beta}(t) = (a \cos t, a \sin t, bt) \) (of Example 3) by means of the formula derived in Exercise I.1.9.

I.1.11(c) Compute the curvature \( k(t) \) for \( \vec{\alpha}(t) = (\cos t, \sin t, e^t) \).