1.2. Space and Time in Prerelativity Physics and in Special Relativity

Note. Spacetime consists of a collection of 4-tuples of the form (t, x, y, z). Such a 4-tuple is an *event*. Classically, time was thought of as absolute (regardless of position or state of motion), making the concept of simultaneity an easy one.

Note. The finiteness of the speed of light restricts which events an observer can see. Consider the light cone in Figure 1.2.

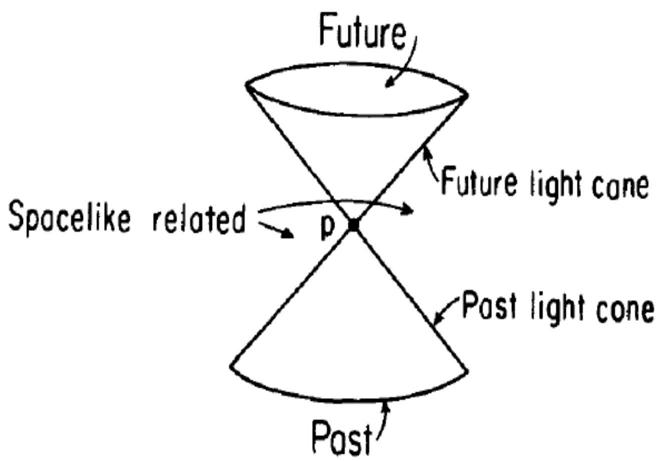

Figure 1.2

In this figure, the time axis is vertical and there are two spatial dimensions represented by horizontal axes. An observer at event p can see the events on the past light cone and observers on the future light cone can see event p. The fact that a particle with rest mass (such as an observer) can only move at a speed less than that of light (a postulate of special relativity) puts a causal relation on the events in spacetime. The events interior to the past light cone are events at which an observer at event p could have been present (and so these are the events which can causally affect the observer at event p). Similarly, the events in the interior of the future light cone are those events at which an observer at point p can be present (and so these are the events which an observer at point p can causally affect). The points exterior to the future and past light cones are points which neither affect event p nor can be affected by event p. If the observer at event p maintain the same spatial coordinates then, as time passes, the observer will be able to see the events exterior to both light cones. The path that an observer follows through spacetime is the *world line* of the observer. The world line of an observer at event p must (as time increases) be contained in the past light cone, pass through event p, and be contained in the future light cone.

Note. Suppose two inertial observers O and O' set up coordinate systems (t, x, y, z) and (t', x', y', z'), respectively. Suppose the event p occurs with coordinates $t = x = y = z = t' = x' = y' = z' = 0$ and that observer O' moves along the x-axis of observer O with velocity v and that the x'-axis and x-axis coincide. Then the relation between (t, x, y, z) and (t', z', y', z') is given by:

$$t' = t, \quad x' = x - vt, \quad y' = y, \quad z' = z.$$

This is called the *Galilean transformation*.

Note. In special relativity, the coordinates of observers O and O' are related as

$$t' = \frac{t - vx/c}{\sqrt{1 - v^2/c^2}}, \quad x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z,$$

where c is the speed of light. This is called the *Lorentz transformation*.