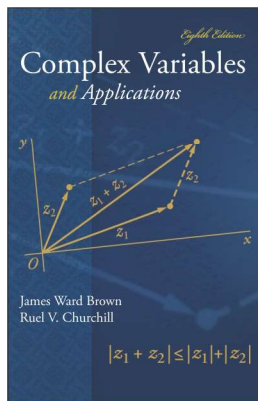


Complex Variables

Chapter 1. Complex Numbers

Section 1.11. Regions in the Complex Plane—Proofs of Theorems



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Complex Variables

February 6, 2020

1 / 4

Lemma 1.11.A

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Lemma 1.11.A. A point z_0 is a boundary point of set S if and only if every ε neighborhood of z_0 contains at least one point in set S and at least one point not in S .

Proof. Suppose z_0 is a boundary point of set S . Then, by definition, z_0 is neither an interior point of S nor an exterior point of S . So z_0 has no neighborhoods which are subsets of S and z_0 has no neighborhoods which are disjoint from S . So each neighborhood of z_0 must contain a point of S and also contain a point not in S , as claimed.

Now suppose every neighborhood of z_0 contains at least one point in set S and at least one point not in S . Then no neighborhood of z_0 is a subset of set S and hence z_0 is not an interior point of S . Also, no neighborhood of z_0 is disjoint with set S and hence z_0 is not an exterior point of S . So, by definition, z_0 is a boundary point of S . \square

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Complex Variables

February 6, 2020

3 / 4

Lemma 1.11.B

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Lemma 1.11.B. If a set $S \subset \mathbb{C}$ is closed, then S contains all of its accumulation points.

Proof. Let z_0 be an accumulation point of set S . ASSUME $z_0 \notin S$. Then each deleted neighborhood of z_0 contains at least one point of S , and so each neighborhood of z_0 contains both a point in S and a point not in S (namely z_0 itself). Hence, by Lemma 1.11.A, z_0 is a boundary point of S . But, by definition, a closed set contains all of its boundary points, a CONTRADICTION. So the assumption that $z_0 \notin S$ is false and it must be that $z_0 \in S$. Since z_0 is an arbitrary accumulation point of S , then S contains all of its accumulation points. \square

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Complex Variables

February 6, 2020

4 / 4