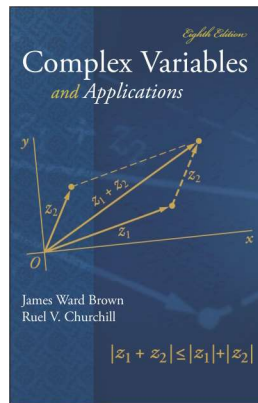


Complex Variables

Chapter 1. Complex Numbers

Section 1.4. Vectors and Moduli—Proofs of Theorems



Corollary 1.4.1

Corollary 1.4.1. For all $z_1, z_2 \in \mathbb{C}$, we have

$$||z_1| - |z_2|| \leq |z_1 + z_2|.$$

Proof. By the Triangle Inequality,

$$|z_1| = |(z_1 + z_2) + (-z_2)| \leq |z_1 + z_2| + |-z_2| = |z_1 + z_2| + |z_2|$$

and so $|z_1| - |z_2| \leq |z_1 + z_2|$. Similarly,

$$|z_2| = |(z_1 + z_2) + (-z_1)| \leq |z_1 + z_2| + |-z_1| = |z_1 + z_2| + |z_1|$$

and so $|z_2| - |z_1| \leq |z_1 + z_2|$. Since $||z_1| - |z_2||$ equals either $|z_1| - |z_2|$ or $|z_2| - |z_1|$, then $||z_1| - |z_2|| \leq |z_1 + z_2|$. \square