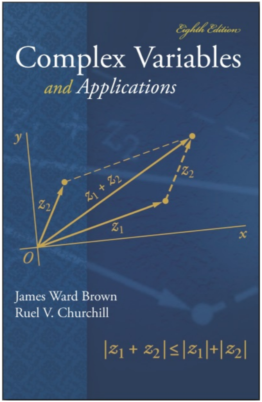
Complex Variables

Chapter 1. Complex Numbers

Section 1.4. Vectors and Moduli—Proofs of Theorems



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Proof. By the Triangle Inequality,

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and so $|z_1| - |z_2| \leq |z_1 + z_2|$.

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and so $|z_2| - |z_1| \leq |z_1 + z_2|$. Since $||z_1| - |z_2||$ equals either $|z_1| - |z_2|$ or $|z_2| - |z_1|$, then $||z_1| - |z_2|| \leq |z_1 + z_2|$. □

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