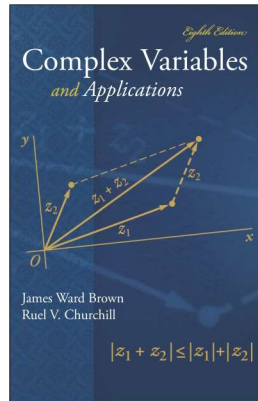


Complex Variables

Chapter 2. Analytic Functions

Section 2.19. Derivatives—Proofs of Theorems



Theorem 2.19.A

Theorem 2.19.A. Differentiable implies Continuous. If f is differentiable at point z_0 then f is continuous at z_0 .

Proof. First, by the definition of derivative, the domain of f contains some open neighborhood of z_0 . Also,

$$\begin{aligned} \lim_{z \rightarrow z_0} (f(z) - f(z_0)) &= \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0} (z - z_0) \\ &= \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0} \lim_{z \rightarrow z_0} (z - z_0) \text{ provided each} \\ &\hspace{15em} \text{constituent limit exists} \\ &= f'(z_0) \cdot 0 = 0. \end{aligned}$$

Therefore $\lim_{z \rightarrow z_0} f(z) - \lim_{z \rightarrow z_0} f(z_0) = 0$ or $\lim_{z \rightarrow z_0} f(z) = \lim_{z \rightarrow z_0} f(z_0) = f(z_0)$. So, by definition, f is continuous at z_0 . \square