Section 1.10. Examples

Example 1.10.1. Find three cube roots of $-8i$. Notice that Brown and Churchill denote the as $(-8i)^{1/3}$, but we should be careful because this should not be confused with the cube root function (since a function is, by definition, single valued).

Solution. First notice that $-8i = 8\exp(i(-\pi/2)) = 8\exp(i(-\pi/2 + 2k\pi))$ for $k \in \mathbb{Z}$. So

$$c_k = \sqrt[3]{8}\exp(i(-\pi/2 + 2k\pi)/3)) = \sqrt[3]{8}\exp(i(-\pi/6 + 2k\pi/3))$$

for $k \in \mathbb{Z}$. We then have

$$\begin{aligned} c_0 &= 2\exp(i(-\pi/6)) = 2(\sqrt{3}/2 + i(-1/2)) = \sqrt{3} - i \\ c_1 &= 2\exp(i(-\pi/6 + 2\pi/3)) = 2\exp(i(\pi/2)) = 2(0 + i(1)) = 2i \\ c_2 &= 2\exp(i(-\pi/6 + 4\pi/3)) = 2\exp(i(7\pi/6)) = 2(-\sqrt{3}/2 + i(-1/2)) = -\sqrt{3} - i. \end{aligned}$$

We then have geometrically that c_0, c_1, and c_2 are as seen in Figure 12.

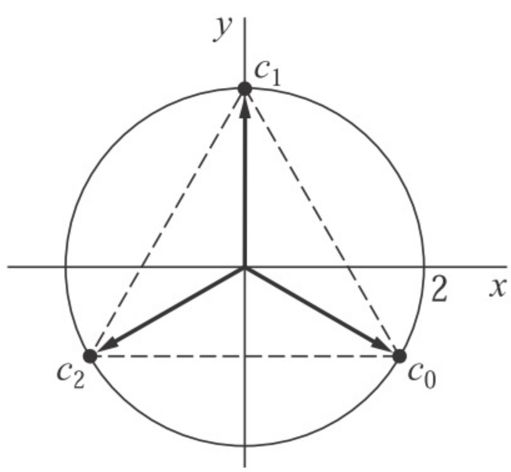

FIGURE 12

Example 1.10.2. We have already stated that the distinct nth roots of unity are $\omega_n^k = s_k = \exp(i(2k\pi/n))$ for $k = 0, 1, \ldots, n-1$. Figure 13 shows the location of the cube roots, 4th roots, and 6th roots of unity.

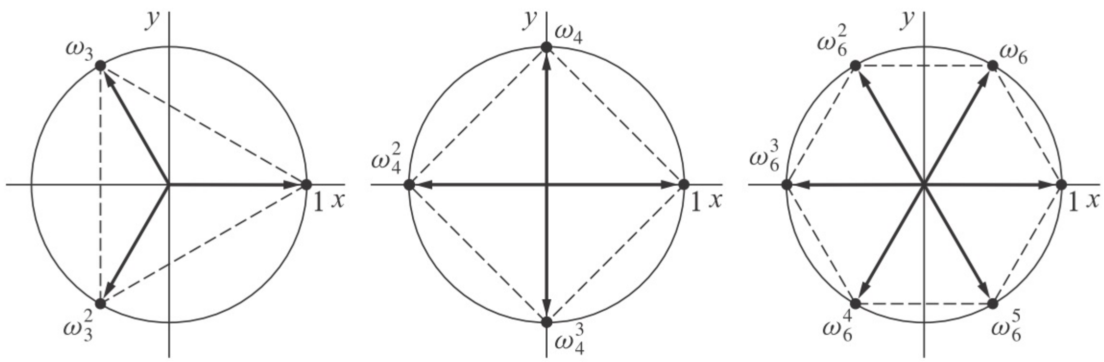

FIGURE 13