# **Section 1.3.** Further Properties

**Note.** In this brief section, we introduce a few formulas (as lemmas) and the Binomial Theorem.

**Lemma 1.3.1.** For any $z_1, z_2, z_3 \in \mathbb{C}$, with $z_3 \neq 0$, we have

$$\frac{z_1}{z_3} + \frac{z_2}{z_3} = \frac{z_1 + z_2}{z_3}.$$

**Note.** Lemma 1.3.1 deals with addition of quotients and "common denominators." The following relates inverses and products ("the products of inverses is the inverse of the products").

**Lemma 1.3.2.** If $z_1, z_2 \in \mathbb{C}$, with $z_1 \neq 0$, $z_2 \neq 0$, then

$$\left(\frac{1}{z_1}\right)\left(\frac{1}{z_2}\right) = \frac{1}{z_1 z_2},$$

or equivalently, $z_1^{-1} z_2^{-1} = (z_1 z_2)^{-1}$.

**Theorem 1.3.2. The Binomial Theorem.**

For any $z_1, z_2, z_3 \in \mathbb{C}$ and $n \in \mathbb{N} = \{1, 2, 3, \ldots\}$ (the natural numbers) we have

$$(z_1 + z_2)^n = \sum_{k=0}^{n} \binom{n}{k} z_1^n z_2^{n-k}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)(n-2)\cdots(n-k+1)!}{k(k-1)(k-2)\cdots(3)(2)(1)}$$

and $0! = 1$ (by definition).

**Note.** You are asked to prove the Binomial Theorem in Exercise 1.3.8 using Mathematical Induction. This is similar to the proof you likely saw in Mathematical Reasoning (MATH 3000).

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