

Section 5.66. Uniqueness of Series Representations

Note. We now use Theorem 5.65.1 to show, for a given function, the uniqueness of a Taylor series representation on a given disk and the uniqueness of a Laurent series on a given annulus.

Theorem 5.66.1. If a series $\sum_{n=0}^{\infty} a_n(z - z_0)^n$ converges to $f(z)$ at all points interior to some circle $|z - z_0| = R$, then it is the Taylor series expansion for f in powers of $z - z_0$.

Theorem 5.66.2. If a series

$$\sum_{n=-\infty}^{\infty} c_n(z - z_0)^n = \sum_{n=0}^{\infty} a_n(z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n}$$

converges to $f(z)$ at all points in some annular domain about z_0 , then it is the Laurent series expansion for f in powers of $z - z_0$ for that domain.

Revised: 12/22/2019