Section 6.73. Residues at Poles

Note. So far we have required a Laurent series to find a residue at a singular point. We now give a theorem which simplifies the procedure for finding residues when dealing with isolated singular points which are poles.

Theorem 6.73.1. An isolated singular point z_0 of a function f has a pole of order m if and only if f can be written in the form $f(z) = \dfrac{\varphi(z)}{(z - z_0)^m}$ where φ is analytic for $|z| < R_2$ for some $R_2 > 0$, and $\varphi(z_0) \neq 0$. Moreover,

$$\text{Res}_{z=z_0} f(z) = \begin{cases} \varphi(z_0) & \text{if } m = 1 \\ \frac{\varphi^{(m-1)}(z_0)}{(m-1)!} & \text{if } m \geq 2. \end{cases}$$

Revised: 4/8/2018