

# Chapter 1. Complex Numbers

## Study Guide

The following is a brief list of topics covered in Chapter 1 of Brown and Churchill's *Complex Variables and Applications*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

### Section 1.1. Sums and Products.

Definition of the field of complex numbers  $\mathbb{C}$  in terms of ordered pairs of real numbers, real and imaginary axes,  $i = (0, 1)$ ,  $i^2 = -1$ , "FOIL" holds in  $\mathbb{C}$ .

### Section 1.2. Basic Algebraic Properties.

Properties of addition and multiplication as given in Theorem 1.2.1,  $\mathbb{C}$  has no zero divisors, computation of  $1/z$ .

### Section 1.3. Further Properties.

Addition with common denominators (Lemma 1.3.1), products of multiplicative inverses (Lemma 1.3.2), the Binomial Theorem (Theorem 1.3.2).

### Section 1.4. Vectors and Moduli.

Modulus, the vector interpretation of complex numbers, the Triangle Inequality, other inequalities involving moduli (Corollary 1.4.1).

### Section 1.5. Complex Conjugates.

Complex conjugate  $\bar{z}$ , properties of conjugates (Theorem 1.5.1), the modulus of products and quotients (Theorem 1.5.2).

### Section 1.6. Exponential Form.

Argument, principal value of the argument, Euler's formula, exponential form  $z = re^{i\theta}$ .

### Section 1.7. Products and Powers in Exponential Form.

Products and quotients in exponential form (Theorem 1.7.1), powers in exponential form (Corollary 1.7.2),  $(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$  (Corollary 1.7.3).

### Section 1.8. Arguments of Products and Quotients.

Arguments of products (Lemma 1.8.1), arguments of quotients (Lemma 1.8.2), finding arguments.

### Section 1.9. Roots of Complex Numbers.

$n$ th roots of unity,  $e^{i\theta}$  is periodic, computing  $n$ th roots, distribution of  $n$ th roots around a regular

$n$ -gon, the principal  $n$ th root.

**Section 1.10. Examples.**

Computation of  $n$ th roots.

**Section 1.11. Regions in the Complex Plane.**

Use of modulus to measure distance, definition of  $\varepsilon$  neighborhood, and deleted  $\varepsilon$  neighborhood, interior point (of a set), exterior point, boundary point, boundary, classification of a boundary point (Lemma 1.11.A), open set, closed set, closure of a set, connected open set, polygonal line, domain, region, bounded/unbounded set, accumulation point, closed sets contain their accumulation points (Lemma 1.11.B), isolated point.

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