Chapter 2. Analytic Functions
Study Guide

The following is a brief list of topics covered in Chapter 2 of Brown and Churchill's *Complex Variables and Applications*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

Section 2.12. Functions of a Complex Variable.
Function, value, domain of definition, codomain, range, polynomial, rational function.

Section 2.13. Mappings.
Image, inverse image, translation, rotation, reflection, examples.

Section 2.14. Mappings of the Exponential Function.
The exponential function in polar form, examples.

Section 2.15. Limits.
Definition of limit, uniqueness of limits (Lemma 2.15.A), definition of limit involving a boundary point of the domain, the Two-Path Test (Note 2.15.A).

Section 2.16. Theorems on Limits.
Definition of limit of a real-valued function of two real variables, relationship of a complex limit to limits of the real and imaginary parts (Theorem 2.16.1), limits of sums, products, and quotients (Theorem 2.16.2), linearity of limits (Note 2.16.A), evaluating limits (Lemma 2.16.A, Lemma 2.16.B, Corollary 2.16.A, Corollary 2.16.B).

Section 2.17. Limits Involving the Point at Infinity.
Stereographic projection, Riemann sphere, extended complex plane, the point at infinity ∞, ε neighborhood of ∞, definition of $\lim_{z\to\infty} f(z) = w_0$, definition of $\lim_{z\to z_0} f(z) = \infty$, definition of $\lim_{z\to\infty} f(z) = \infty$, computations of limits involving ∞ (Theorem 2.17.1).

Section 2.18. Continuity.
Definition of continuity at a point and on a region, continuity of sums, differences, products, and quotients of continuous functions (Corollary 2.18.A), continuity of polynomial functions (Corollary 2.18.B), continuity of compositions (Theorem 2.18.1), property of continuous functions (Theorem 2.18.2), extreme value theorem (Theorem 2.18.3).

Section 2.19. Derivatives.

Definition of derivative, differentiable function, examples, differentiable implies continuous (Theorem 2.19.A).

Section 2.20. Differentiation Formulas.

Linearity of derivatives (Theorem 2.20.A), Product and Quotient Rules (Theorem 2.20.B), derivatives of z^n for $n \in \mathbb{N}$ and $n \in \mathbb{Z}$ (Corollaries 2.20.A and 2.20.B), the Chain Rule (Theorem 2.20.C), "square bracket notation."

Section 2.21. Cauchy-Riemann Equations.

The Cauchy-Riemann equations (Theorem 2.21.A), examples.

Section 2.22. Sufficient Conditions for Differentiability.

The Cauchy-Riemann Equations and Continuity Imply Differentiability (Theorem 2.22.A), examples.

Section 2.23. Polar Coordinates.

Cauchy-Riemann equations in polar coordinates (Lemma 2.23.A), Cauchy-Riemann equations in polar coordinates imply Cauchy-Riemann equations in rectangular coordinates (Lemma 2.23.B), the derivative in polar coordinates (Lemma 2.23.C).

Section 2.24. Analytic Functions.

"Analytic" really means power series, definition of analytic at a point and on a set, entire function, singular point, linear combinations and quotients of analytic functions (Lemma 2.24.A), compositions of analytic functions (Lemma 2.24.B), functions with derivative 0 are constant (Theorem 2.24.A).

Section 2.25. Examples.

$f(z)$ and $\overline{f(z)}$ analytic implies f is constant (Theorem 2.25.A), with $f(z)$ analytic $|f(z)|$ constant implies f constant (Theorem 2.25.B), analytic $f(z)$ real-valued implies f constant (Theorem 2.25.C).

Section 2.26. Harmonic Functions.

Harmonic function, Laplace's equation, harmonic (real) functions make analytic (complex) functions (Theorem 2.26.1), harmonic conjugate, harmonic conjugates and analytic functions (Theorem 2.26.2), examples.

Section 2.27. Uniquely Determined Analytic Functions.

Analytic functions which must be constant (Lemma 2.27.A), uniquely determined analytic function (Theorem 2.27.A), an infinitely differentiable function of a real variable which has no power series representation.

Section 2.28. Reflection Principle.
The Reflection Principle (Theorem 2.28.A), polynomials, e^z, and the trigonometric functions satisfy the Reflection Principle.

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