Chapter 3. Elementary Functions
Study Guide

The following is a brief list of topics covered in Chapter 3 of Brown and Churchill's *Complex Variables and Applications*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

Section 3.29. The Exponential Function.

Multiplying exponentials (Lemma 3.29.A), dividing exponentials.

Section 3.30. The Logarithm Function.

The "multiple-valued" function $\log z$, the principal value $\mathrm{Log}\, z$, relationship between $\log z$ and $\mathrm{Log}\, z$.

Section 3.31. Branches and Derivatives of Logarithms.

$\log z$ as an analytic function, branch of the logarithm, branch cut, branch point.

Section 3.32. Some Identities Involving Logarithms.

Logarithm of a product (Lemma 3.32.A), logarithm of a quotient (Lemma 2.32.B), $z^n = e^{n \log z}$ (Lemma 3.32.C), nth roots in terms of logarithms (Lemma 3.32.D).

Section 3.33. Complex Exponents.

Definition of z^c where $c \in \mathbb{C}$, branch of z^c, principal branch of z^c, principal value of z^c, derivative of z^c (Theorem 3.33.A), examples, exponential function base c: c^z.

Section 3.34. Trigonometric Functions.

Definition of $\cos z$ and $\sin z$, derivatives of $\sin z$ and $\cos z$, addition formulas for sin and cos, double angle formulas, cofunction relationship between $\sin z$ and $\cos z$, real and imaginary parts of $\sin z$ and $\cos z$ (Lemma 3.34.A), the modulus of $\sin z$ and $\cos z$ and the fact that these are unbounded functions, zeros of a function, roots of a polynomial, zeros of $\sin z$ and $\cos z$ (Lemma 3.34.B), definitions and derivatives of the other trigonometric functions.

Section 3.35. Hyperbolic Functions.

Definition of $\cosh z$ and $\sinh z$, derivatives of $\cosh z$ and $\sinh z$ (Note 3.35.A), identities (Note 3.35.B), zeros of $\cosh z$ and $\sinh z$ (Note 3.35.C), definition and derivatives of the other hyperbolic trig functions.

Section 3.36. Inverse Trigonometric and Hyperbolic Functions.

Definition of $\sin^{-1} z$ as a "multiple-valued function," definitions of $\cos^{-1} z$ and $\tan^{-1} z$ as "multiple-valued functions," derivatives of inverse trig and inverse hyperbolic trig functions.

Revised: 1/15/2020