Chapter 4. Integrals

Study Guide

The following is a brief list of topics covered in Chapter 4 of Brown and Churchill's *Complex Variables and Applications*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

Section 4.37. Derivatives of Functions $w(t)$.
Path $w(t)$, $\dfrac{d}{dt}[z_0 w(t)]$, $\dfrac{d}{dt}[e^{z_0 t}]$, the Mean Value Theorem need not hold for $w(t)$.

Section 4.38. Definite integrals of Functions $w(t)$.
Definition of the integral of $w(t)$, examples.

Section 4.39. Contours.
Arc, simple arc, simple closed curve, positively oriented simple closed curve, differentiable arc, length of an arc, smooth arc, contour, simple closed contour.

Section 4.40. Contour Integrals.
Definition of contour integral (or line integral), contour $-C$ and its relationship to integrals, contour $C_1 + C_2$ and its relationship to integrals.

Section 4.41. Some Examples.
Examples.

Section 4.42. Examples with Branch Cuts.
Examples involving $z^{1/2}$ and z^{a-1}.

Section 4.43. Upper Bounds for Moduli of Contour Integrals.
Inequality for $\int_a^b w(t)\,dt$ (Lemma 4.43.A), upper bound on a contour integral (Theorem 4.43.A), examples.

Section 4.44. Antiderivatives.
Existence of antiderivatives and their use in evaluating contour integrals (Theorem 4.44.A), examples.

Section 4.45. Proof of the Theorem.
Proof of Theorem 4.44.A.

Section 4.46. Cauchy-Goursat Theorem.

Green's Theorem (from vector calculus), integrals of an analytic function with continuous derivative around a closed contour is 0, Cauchy-Goursat Theorem (Theorem 4.46.A)

Section 4.47. Proof of the Theorem.

Proof of the Cauchy-Goursat Theorem (Theorem 4.44.A) using Lemma 4.47.1 and Lemma 4.47.A.

Section 4.48. Simply Connected Domains.

Simply connected domain, integrals over closed contours in a simply connected domain are 0 (Theorem 4.48.A), antiderivatives of analytic functions on simply connected domains (Corollary 4.48.B).

Section 4.49. Multiply Connected Domains.

Multiply connected domain, integrals over several contours (Theorem 4.49.A), Principle of Deformation (Corollary 4.49.B), homotopic contours over a region.

Section 4.50. Cauchy Integral Formula.

Cauchy Integral Formula (Theorem 4.50.A).

Section 4.51. An Extension of the Cauchy Integral Formula.

General Cauchy Integral Formula (Theorem 4.55.A), examples.

Section 4.52. Some Consequences of the Extension.

f analytic at a given point implies that all of the derivatives of f are analytic at that point (Theorem 4.52.1), real and imaginary parts of an analytic functions have continuous partials of all orders (Corollary 4.52.A), Morera's Theorem (Theorem 4.52.2), Cauchy's Inequality (Theorem 4.52.3).

Section 4.53. Liouville's Theorem and the Fundamental Theorem of Algebra.

Liouville's Theorem (Theorem 4.53.1), the Fundamental Theorem of Algebra (Theorem 4.53.2), other proofs of the Fundamental Theorem of Algebra.

Section 4.54. Maximum Modulus Principle.

Gauss's Mean Value Theorem (Theorem 4.54.B), the Maximum Modulus Theorem (Theorem 4.54.C), Alternative Version of the Maximum Modulus Theorem (Theorem 4.54.D), maximum modulus occurs on the boundary (Theorem 4.54.E), the Minimum Modulus Theorem (Corollary 4.54.F), the Maximum Modulus Theorem for Unbounded Domains (Theorem 4.54.G).

Supplement. Applications of the Maximum Modulus Theorem to Polynomials.

Centroid, The Centroid Theorem, half planes represented as inequalities, the Gauss-Lucas Theorem, The Lucas Corollary, the Eneström-Kakeya Theorem, Joyal/Labelle/Rahman Theorem, Gardner/Govil Theorem, Rate of Growth Theorem, Ankeny/Rivlin Theorem, Aziz/Dawood Theorem, sup norm $\|p\|_\infty$, Bernstein Lemma, Bernstein's Inequality, Erdös-Lax Theorem, de Bruij's Theorem and $\|p\|_\delta$.

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