Chapter 5. Series

Study Guide

The following is a brief list of topics covered in Chapter 5 of Brown and Churchill's *Complex Variables and Applications*, 8th edition. This list is not meant to be comprehensive, but only gives a list of several important topics. You should also carefully study the examples and proofs given in class and in the homework problems.

Section 5.55. Convergence of Sequences.
Sequence, limit of a sequence, convergent/divergent sequence, relationship between the limit of a sequence and the limits of the real and imaginary parts (Theorem 5.55.A).

Section 5.56. Convergence of Series.
Series, sum of a series, convergent/divergent series, relationship between the sum of a series and the sums of the real and imaginary parts (Theorem 5.56.A), Test for Divergence (Corollary 5.56.1), absolutely convergent series, absolute convergence implies convergence (Corollary 5.56.2), partial sum and remainder, power series and coefficients, the geometric series $\sum_{n=0}^{\infty} z^n$.

Section 5.57. Taylor Series.
Taylor's Theorem (Theorem 5.57.A), Taylor series, Maclaurin series, comparison of real and complex differentiability.

Section 5.58. Proof of Taylor's Theorem.
Proof of Taylor's Theorem (Theorem 5.57.A), observation on the roll played by geometric series in the proof.

Section 5.59. Examples.
Power series for e^z, $\sin z$, $1/(1-z)$, and a rational function.

Section 5.60. Laurent Series.
Laurent's Theorem (Theorem 5.60.1), Laurent series and the coefficients as integrals.

Section 5.61. Proof of Laurent's Theorem.
Proof of Laurent's Theorem.

Section 5.62. Examples.
Examples showing the computation of Laurent series.

Section 5.63. Absolute and Uniform Convergence of Power Series.

Absolute convergence of power series (Theorem 5.63.1), circle of convergence, radius of convergence, definition of uniform convergence, uniform convergence of power series on closed discs (Theorem 5.63.2).

Section 5.64. Continuity of Sums of Power Series.

Power series represent continuous functions (Theorem 5.64.1), absolute convergence of Laurent series (Note 5.64.A).

Section 5.65. Integration and Differentiation of Power Series.

Term-by-term integration (Theorem 5.65.1), power series represent analytic function (Corollary 5.65.1), a function is analytic if and only if it has a power series representation, differentiation of power series term-by-term (Theorem 5.65.2).

Section 5.66. Uniqueness of Series Representations.

A power series representation must be the Taylor series (Theorem 5.66.1), a double series representation must be the Laurent series (Theorem 5.66.2).

Section 5.67. Multiplication and Division of Power Series.

The product of two series (the Cauchy product), examples.

Revised: 1/2/2020