Lemma 3.9.1. Let $A$ be a locally finite collection of subsets of $X$. Then:

\[ A \subseteq V \Rightarrow A \subseteq V \]

Proof (continued). (c) Denote $V = \bigwedge A$. Now each $A \in A$ is a

\[ A \cup A \subseteq V \]

since $X$ is metrizable, there is a metric $d$ on $X$. Let $u \in N$. Given $u \in A$,

there is an open covering of $X$ that is a $d$-neighborhood of $u$. Then by

\[ u \in A \]

let $U$ be a $d$-neighborhood of $x$. Then, since $A$ is locally finite in $X$, $U$

intersects only finitely many elements of $A$. Say $A_1, A_2, \ldots, A_k$. Assume

\[ x \notin A_1 \cup A_2 \]

but $x \in A_k$. Then $\bigwedge A_k \subseteq V$. Therefore $A \subseteq V$.

Chapter 6. Metrization Theorems and Paracompactness

Section 39. Local Finiteness—Proofs of Theorems

Introduction to Topology
Lemma

Lemma 39.2 (continued 4)

[Image 403x526 to 498x658]

Lemma 39.2 (continued 3)

[Image 313x82 to 442x262]