Section 42. The Smirnov Metrization Theorem

Note. Recall that the Nagata-Smirnov Metrization Theorem (theorem 40.3) states that a space is metrizable if and only if it is regular and has a basis that is countably locally finite. In this section we give another necessary and sufficient condition for metrizability, this time involving paracompactness. First, we need a definition.

Definition. A space \( X \) is locally metrizable if every point \( x \in X \) has a neighborhood \( U \) that is metrizable in the subspace topology.

Note. Of course, a metrizable space is locally metrizable.

Theorem 42.1. The Smirnov Metrization Theorem.
A topological space \( X \) is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable.

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