Complex Analysis 1, Test 1 Study Guide

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The Meaning of Mathematics. Formalism, *Principia Mathematica*, Russell's Paradox, Hilbert, Frege, Russell, Gödel, WFF, completeness, consistence, misapplications of Gödel's work, Peano's axioms.

1.1 The Real Numbers. Complete ordered field, uniqueness of a complete ordered field.

1.2 The Field of Complex Numbers. Definition of $\mathbb{C}$, "Is $\mathbb{C}$ isomorphic to $\mathbb{R}^2$?" modulus, conjugate.

1.3 The Complex Plane. Geometric relationship between $\mathbb{R}^2$ and $\mathbb{C}$, Triangle Inequality and its sharpness, Cauchy sequences of real numbers, Axiom of Completeness.

Ordering the Complex Numbers. Ordered fields, Law of Trichotomy, Corollaries 1 and 3 (i is not positive, $-i$ is not positive), Theorem 3 ($\mathbb{C}$ is not an ordered field), lexicographic ordering of $\mathbb{C}$ and its uselessness, well ordering, total ordering, Well-Ordering Principle.

1.4 Polar Representations and Roots of Complex Numbers. Argument, $\text{cis}(\theta)$, DeMoivre's Formula, nth roots of unity.

1.5 Lines and Half-Planes in the Complex Plane. Equation for a line, half planes as inequalities.

Ilieff-Sendov Conjecture. Gauss-Lucas Theorem, Corollary 1 (convex polygon containing zeros of a polynomial), Corollary 2 (circle containing zeros), Theorem 2 (centroid of zeros), Ilieff-Sendov Conjecture.

1.6 The Extended Plane and Its Spherical Representation. Extended plane, Riemann sphere, stereographic projection, projections of circles.

3.1 Power Series. Absolute convergence, lim inf/lim sup, power series, geometric series, radius of convergence, Ratio Test.

3.2 Analytic Functions. Differentiable at a point, Differentiable implies Continuous, analytic, Chain Rule, Proposition III.2.5, e^z, $\cos z$, $\sin z$, periodic function, branch of the log, principla branch of log, branch of z^b, region, Cauchy Riemann Equations, harmonic function, harmonic conjugate.

A Primer on Lipschitz Functions. Derivative of a function between two metric spaces, Lipschitz and locally Lipschitz functions between two metric spaces, examples of functions showing converse statements do not hold, C^n functions, Lip^n functions.