Complex Analysis 1, Test 1 Study Guide

Prepared by Dr. Robert Gardner

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The Meaning of Mathematics. Formalism, *Principia Mathematica*, Russell's Paradox, Hilbert, Frege, Russell, Gödel, WFF, completeness, consistence, misapplications of Gödel's work, Peano's axioms.

1.1 The Real Numbers. completeness

1.2 The Field of Complex Numbers. Definition of $\mathbb{C}$, modulus, conjugate, uniqueness of $\mathbb{R}$.

Ordering the Complex Numbers. Ordered fields, Law of Trichotomy, Corollaries 1 and 3 (i is not positive, $-i$ is not positive), Theorem 3 ($\mathbb{C}$ is not an ordered field), lexicographic ordering of $\mathbb{C}$ and its uselessness, well ordering, total ordering, Well-Ordering Principle.

1.3 The Complex Plane. Geometric relationship between $\mathbb{R}^2$ and $\mathbb{C}$, Triangle Inequality and its sharpness.

1.4 Polar Representations and Roots of Complex Numbers. Argument, $\text{cis}(\theta)$, DeMoivre's Formula, nth roots of unity, primitive roots.

1.5 Lines and Half-Planes in the Complex Plane. Equation for a line, half planes as inequalities.

1.6 The Extended Plane and Its Spherical Representation. Extended plane, Riemann sphere, stereographic projection.

Ilieff-Sendov Conjecture. Gauss-Lucas Theorem, Corollary 1 (convex polygon containing zeros of a polynomial), Corollary 2 (circle containing zeros), Theorem 2 (centroid of zeros), Ilieff-Sendov Conjecture.

2.1 Definition and Examples of Metric Spaces. Metric space, balls, taxicab/usual metrics, ℓ^2, L^2, open/closed sets and properties, topological space, interior/closure/boundary, Theorem 1.13 (relationships between interior/closure/boundary), dense set, countable dense sets.

2.2 Connectedness. Connected, connectedness in a topological space, polygon in $\mathbb{C}$, Theorem 2.3 (polygons and connectedness for open sets in $\mathbb{C}$), component, Theorem 2.7 (components partition a space), Theorem 2.9 (components of an open set in $\mathbb{C}$).

2.3 Sequences and Completeness. Convergence, convergence in a topological space and nonuniqueness of limits, limit point and Theorem 3.4, Cauchy, complete metric space, Theorem 3.5 ($\mathbb{C}$ is complete), diameter of a set, Theorem 3.8 ("Cantor's Theorem").

2.4 Compactness. Definition, Heine-Borel Theorem, example of a closed and bounded set which is not compact (in ℓ^2), Theorem 4.3 (compact implies closed, closed subset of compact is compact).

2.5 Continuity. Definition of limit of a function between two metric spaces, definition of continuity, Theorems 5.2 and 5.3 (properties equivalent to continuity), continuity of a function between two topological spaces, uniform continuity, Lipschitz, contraction, distance between sets, Theorem 5.7 (relationships between distance closure, and limit points).

A Primer on Lipschitz Functions. Derivative of a function between two metric spaces, Lipschitz and locally Lipschitz functions between two metric spaces, examples of functions showing converse statements do not hold, C^n functions, Lip^n functions.